Closing Tues: HW 13.4, 14.1
Closing Thur: HW 14.2 (part 1)

14.1/14.2 Partial Derivatives

Goal: To find derivatives of multivariable functions.

Idea: Look at one variable at a time.

Entry Task: Consider

$$f(x, y) = 4xy + y^2 - 3x - 5y$$

1. Plug in y = 1, then find the derivative with respect to x.
2. Do it again with y = 2, …
3. And again with y = 3, …

4. Plug in x = 1, then find the derivative with respect to y.
5. Do it again with x = 2, …
6. And again with x = 3, …

Recall: Definition of derivative

1. Given a function $y = f(x)$.
2. Simplify the general formula for the slope of the secant from x to $x + h$

$$\frac{f(x+h) - f(x)}{h}$$

3. Let $h \to 0$, to get

$$\frac{dy}{dx} = f'(x) = \text{slope of tangent}$$

Example:

$$f(x, y) = 4xy + y^2 - 3x - 5y$$

$$\frac{f(x+h, y) - f(x, y)}{h} =$$

Partial Derivatives

For multivariable functions:

1. Given $z = f(x, y)$

2a. Simplify (y fixed, x variable)

$$\frac{f(x+h, y) - f(x, y)}{h}$$

3a. Let $h \to 0$, to get

$$\frac{\partial z}{\partial x} = f_x(x, y) \qquad \text{(with respect to x)}$$

2b. Simplify (x fixed, y variable)

$$\frac{f(x, y+h) - f(x, y)}{h}$$

3b. Let $h \to 0$, to get

$$\frac{\partial z}{\partial y} = f_y(x, y) \qquad \text{(with respect to x)}$$

Example:

$$f(x, y) = 4xy + y^2 - 3x - 5y$$

$$\frac{f(x, y+h) - f(x, y)}{h} =$$

How to do partial derivatives:

Step 0: Rewrite powers and simplify like we always do.

Step 1: Identify the desired variable! (Underline it if it helps)
Treat all other variable like numbers!

Step 2: Identify the constants terms and the coefficients.
"Bring down coefficients"

Step 3: Use the regular one-variable derivative rules.

Example: Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for

1. $z = 10x^4 + 7xy^3 + 8x^2y^{10}$

More examples: Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for

1. $z = x^3 - y^2 + 3xy^4$

2. $z = e^{x^2} - \ln(y) + 7$

3. $z = (x^2 + 3y)^{10}$

4. $z = xy^2e^x$

Interpreting as a rate

Your company produces and sells **two** products (hats and sunglasses)

$x = \text{number of hats}$

$y = \text{number of glasses}$

You find that profit is given by

$$P(x, y) = -3x^2 + 30x - 5y^2 + 130y + 2xy - 100$$

1. Find the partial derivatives.
2. Find and interpret $P_x(5,8)$ and $P_y(5,8)$.
3. Estimate the values of

$$\frac{P(5.001,8) - P(5,8)}{0.001} \approx$$

$$\frac{P(5, 8.01) - P(5,8)}{0.01} \approx$$

Graphical Interpretation

Pretend you are skiing on the surface

$z = f(x, y) = 15 - x^2 - y^2$

1. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

2. Find and interpret

   $f_x(7,4)$ and $f_y(7,4)$

Aside: Graphical Interpretations